Option-Implied Equity Premium Predictions via Entropic Tilting

Online Appendix—Not for Publication

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A Gibbs Sampler for the Baseline SV Model

Let $h^t = \{h_1, ..., h_t\}$ be the time-varying log volatility sequence up to time t. Additionally, let $\Theta = \left(\mu, \beta, \sigma_\xi^{-2}, \lambda_0, \lambda_1, \lambda_2\right)$ be the collection of time-invariant parameters of the model. Using $\mathcal{D}^t$ to denote the information set at time t, we obtain draws from the joint posterior distribution $p\left(\Theta, h^t | \mathcal{D}^t\right)$ for the baseline stochastic-volatility (SV) model using the Gibbs sampler to draw recursively from the following conditional distributions:[1]

1. $p\left(\mu, \beta | \, \Theta_{-\mu,\beta}, h^t, \mathcal{D}^t\right)$
2. $p\left(h^t \middle| \, \Theta, \mathcal{D}^t\right)$
3. $p\left(\sigma_\xi^{-2} \middle| \, \Theta_{-\sigma_\xi^{-2}}, h^t, \mathcal{D}^t\right)$
4. $p\left(\lambda_0, \lambda_1, \lambda_2 | \, \Theta_{-\lambda_0,\lambda_1,\lambda_2}, h^t, \mathcal{D}^t\right)$

We simulate draws from each of these blocks as follows. First, conditional on h^t we draw μ and β from standard distributions for $p\left(\mu, \beta | \Theta_{-\mu,\beta}, h^t, \mathcal{D}^t\right)$

$$\left. \begin{bmatrix} \mu \\ \beta \end{bmatrix} \right| \Theta_{-\mu,\beta}, h^t, \mathcal{D}^t \sim \mathcal{N}\left(\overline{b}, \overline{V}\right), \tag{A-1}$$

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[1] In standard set notation, A_{-b} is the complementary set of b in A; $A_{-b} = \{x \in A : x \notin b\}$.

where

$$\begin{aligned}
\overline{V} &= \left[\underline{V}^{-1} + \sum_{\tau=1}^{t-1} \frac{1}{\exp\left(h_{\tau+1}\right)^2} x_\tau x_\tau'\right]^{-1}, \\
\bar{b} &= \overline{V}\left[\underline{V}^{-1}\underline{b} + \sum_{\tau=1}^{t-1} \frac{1}{\exp\left(h_{\tau+1}\right)^2} x_\tau \left(r_{\tau+1} - \mu - \beta' x_\tau\right)\right]. \qquad \text{(A-2)}
\end{aligned}$$

Next, we write equation (8) in the paper as

$$r_{\tau+1}^{*} = \exp\left(h_{\tau+1}\right) u_{\tau+1}, \qquad \text{(A-3)}$$

where $r_{\tau+1}^{*} = r_{\tau+1} - \mu - \beta' x_\tau$. Squaring and taking logs on both sides of (A-3) yields a new state-space system that replaces equations (8) and (9) in the paper with the following

$$\begin{aligned}
r_{\tau+1}^{**} &= 2h_{\tau+1} + u_{\tau+1}^{**}, \qquad \text{(A-4)} \\
h_{\tau+1} &= \lambda_0 + \lambda_1 h_\tau + \lambda_2 RV_\tau + \xi_{\tau+1}, \qquad \text{(A-5)}
\end{aligned}$$

where $r_{\tau+1}^{**} = \ln\left[\left(r_{\tau+1}^{*}\right)^2\right]$, and $u_{\tau+1}^{**} = \ln\left(u_{\tau+1}^2\right) \sim \ln\left(\chi_1^2\right)$, with u_τ^{**} independent of ξ_s for all τ and s.

Kim, Shephard, and Chib (1998) employ a data-augmentation approach and introduce a new state variable s_τ, $\tau = 1, ..., t$, focusing on draws from $p\left(h^t | \Theta, \mathcal{D}^t, s_\tau\right)$ instead of $p\left(h^t | \Theta, \mathcal{D}^t\right)$, and Chib, Nardari, and Shephard (2002) extended further the approach to allow for exogenous covariates in the volatility equation. Conditional on the state variable s_τ, the linear non-Gaussian state-space representation in (A-4) and (A-5) can be written as an approximate linear Gaussian state-space model

$$u_{\tau+1}^{**} \approx \sum_{j=1}^{7} q_j \mathcal{N}\left(m_j - 1.2704, v_j^2\right), \qquad \text{(A-6)}$$

where m_j, v_j^2, and q_j, $j = 1, 2, ..., 7$, are constants specified in Kim et al.. In turn, (A-6) implies

$$u_{\tau+1}^{**}\Big| \, s_{\tau+1} = j \sim \mathcal{N}\left(m_j - 1.2704, v_j^2\right), \qquad \text{(A-7)}$$

where $q_j = \Pr\left(s_{\tau+1} = j\right)$ is the probability of state j.

Conditional on $s^t = \{s_1, s_2, ..., s_t\}$, we can rewrite the nonlinear state-space system as follows

$$\begin{aligned}
r_{\tau+1}^{**} &= 2h_{\tau+1} + e_{\tau+1}, \\
h_{\tau+1} &= \lambda_0 + \lambda_1 h_\tau + \lambda_2 RV_\tau + \xi_{\tau+1}, \qquad \text{(A-8)}
\end{aligned}$$

where $e_{\tau+1} \sim \mathcal{N}\left(m_j - 1.2704, v_j^2\right)$ with probability q_j. The algorithm of Carter and Kohn (1994), modified to allow for the presence of exogenous variables in the volatility equation (Chib, Nardari, and Shephard, 2002), enables us to draw the whole sequence of stochastic volatilities, h^t, for this linear Gaussian state space-system.

Conditional on the sequence h^t, draws of states s^t can be easily obtained noting that each of its elements can be independently drawn from the discrete density defined by

$$\Pr\left(s_{\tau+1} = j \middle| r^{**}_{\tau+1}, h_{\tau+1}\right) = \frac{q_j f_{\mathcal{N}}\left(r^{**}_{\tau+1} \middle| 2h_{\tau+1} + m_j - 1.2704, v_j^2\right)}{\sum_{l=1}^{7} q_l f_{\mathcal{N}}\left(r^{**}_{\tau+1} \middle| 2h_{\tau+1} + m_l - 1.2704, v_l^2\right)} \tag{A-9}$$

for $\tau = 1, ..., t-1$ and $j = 1, ..., 7$, with $f_{\mathcal{N}}$ being the kernel of a normal density.

Third, the posterior distribution for $p\left(\sigma_\xi^{-2} | \Theta_{-\sigma_\xi^{-2}}, h^t, \mathcal{D}^t\right)$ is

$$\sigma_\xi^{-2} \Big| \Theta_{-\sigma_\xi^{-2}}, h^t, \mathcal{D}^t \sim \mathcal{G}\left(\left[\frac{\sum_{\tau=1}^{t-1}\left(h_{\tau+1} - \lambda_0 - \lambda_1 h_\tau - \lambda_2 RV_\tau\right)^2 + \underline{k}_\xi \underline{v}_\xi\left(t_0 - 1\right)}{(t-1) + \underline{v}_\xi\left(t_0 - 1\right)}\right]^{-1}, (t-1) + \underline{v}_\xi\left(t_0 - 1\right)\right). \tag{A-10}$$

Finally, the distribution $p\left(\lambda_0, \lambda_1, \lambda_2 | \Theta_{-\lambda_0,\lambda_1,\lambda_2}, h^t, \mathcal{D}^t\right)$ takes the form

$$\lambda_0, \lambda_1, \lambda_2 | \Theta_{-\lambda_0,\lambda_1,\lambda_2}, h^t, \mathcal{D}^t \sim \mathcal{N}\left(\begin{bmatrix} \overline{m}_{\lambda_0} \\ \overline{m}_{\lambda 1} \\ \overline{m}_{\lambda 2} \end{bmatrix}, \overline{V}_\lambda\right) \times \lambda_1 \in (-1, 1),$$

where

$$\overline{V}_\lambda = \left\{\begin{bmatrix} \underline{V}_{\lambda_0}^{-1} & 0 & 0 \\ 0 & \underline{V}_{\lambda_1}^{-1} & 0 \\ 0 & 0 & \underline{V}_{\lambda_2}^{-1} \end{bmatrix} + \sigma_\xi^{-2} \sum_{\tau=1}^{t-1} \begin{bmatrix} 1 \\ h_\tau \\ RV_\tau \end{bmatrix} [1, h_\tau, RV_\tau]\right\}^{-1}, \tag{A-11}$$

and

$$\begin{bmatrix} \overline{m}_{\lambda_0} \\ \overline{m}_{\lambda 1} \\ \overline{m}_{\lambda 2} \end{bmatrix} = \overline{V}_\lambda \left\{\begin{bmatrix} \underline{V}_{\lambda_0}^{-1} & 0 & 0 \\ 0 & \underline{V}_{\lambda_1}^{-1} & 0 \\ 0 & 0 & \underline{V}_{\lambda_2}^{-1} \end{bmatrix} \begin{bmatrix} \underline{m}_{\lambda_0} \\ \underline{m}_{\lambda_1} \\ \underline{m}_{\lambda_2} \end{bmatrix} + \sigma_\xi^{-2} \sum_{\tau=1}^{t-1} \begin{bmatrix} 1 \\ h_\tau \\ RV_\tau \end{bmatrix} h_{\tau+1}\right\}. \tag{A-12}$$

Using these results, draws from the predictive density $p\left(r_{t+1} | \mathcal{D}^t\right)$ can be obtained by noting that

$$\begin{aligned} p\left(r_{t+1} \middle| \mathcal{D}^t\right) &= \int p\left(r_{t+1} \middle| h_{t+1}, \Theta, h^t, \mathcal{D}^t\right) \times p\left(h_{t+1} \middle| \Theta, h^t, \mathcal{D}^t\right) \\ &\quad \times p\left(\Theta, h^t \middle| \mathcal{D}^t\right) d\Theta dh^{t+1}. \end{aligned} \tag{A-13}$$

Draws from $p\left(r_{t+1}\middle|\mathcal{D}^t\right)$ are obtained in three steps:

1. Draw from $p\left(\Theta, h^t\middle|\mathcal{D}^t\right)$ using the Gibbs sampler described above
2. Simulate the future volatility, h_{t+1} using

$$h_{t+1}|\,\Theta, h^t, \mathcal{D}^t \sim \mathcal{N}\left(\lambda_0 + \lambda_1 h_t + \lambda_2 RV_t, \sigma_\xi^2\right). \tag{A-14}$$

3. Finally, given h^{t+1}, Θ, and $\mathcal{D}^t$, future excess returns are normally distributed

$$r_{t+1}|\,h_{t+1}, \Theta, h^t, \mathcal{D}^t \sim \mathcal{N}\left(\mu + \beta' x_t, \exp\left(h_{t+1}\right)\right). \tag{A-15}$$

B SV Model: Additional Results

B.1 Variance Risk Premium

Figure B-1 shows three different variance measures that are relevant for our tilting exercise. The first is the variance associated with the baseline predictive densities (baseline) for the 15 predictors considered.[2] The second is the end-of-month squared VIX, which is our measure of the risk-neutral variance (risk-neutral).[3] The third is the variance associated with the tilted predictive densities (tilted). Based on a casual look at the various panels, we see that the baseline variances series are highly similar across predictors with correlations exceeding 0.95. A similar point can be made for the tilted variance series. Furthermore, a notable feature of the baseline (tilted) variance is that it exceeds (is smaller) than the risk neutral variance during the entire out-of-sample (OOS) period with a few exceptions across all predictors.

Figure B-2 provides the slope estimate from the variance-risk-premium regression given by equation (22) in the paper, along with 95% confidence intervals, by predictor. The regressions use an expanding window with initial size of 48 monthly observations between January 1986 and December 1989. The slope estimate fails to be statistically different from 1 for the better part of the 25-year period for all predictors, which implies that the variance risk premium equals approximately the average difference between the squared forecast error and the squared VIX. For several predictors (e.g., BM and NTIS) the slope estimate is statistically different from 1 during the last two years or so of the OOS period.

[2] The predictor nomenclature is as follows: (1) DP: Log dividend price-ratio; (2) DY: Log dividend yield; (3) EP: Log earning-price ratio; (4) DE: Log dividend-payout ratio; (5) RVOL: Excess stock return volatility; (6) BM: Book-to-market ratio; (7) NTIS: Net equity expansion; (8) TBL: Treasury bill rate; (9) LTY: Long-term yield; (10) LTR: Long-term return; (11) TMS: Term spread; (12) DFY: Default yield spread; (13) DFR: Default return spread; (14) INFL: Inflation; (15) SII: Short interest index.

[3] This risk-neutral variance series exhibits no variation across the panels of Figure B-1.

Figure B-3 plots the evolution of the estimated variance risk premium in the OOS period annualized and expressed in percentages. For all predictors considered, our estimated variance risk premium is high during periods of economic uncertainty, such as the recent financial crisis (Fall 2008), around the WorldCom and Enron scandals (Summer/Fall 2002) and Gulf War II (Spring 2003), as well as close the Russian financial crisis (Fall 1998). In addition, the same figure shows that our variance risk premium is highly comparable to the variance risk premium from Bollerslev, Tauchen, and Zhou (2009) with the two series tracking each other closely. The only notable deviation between the two variance risk premium series is in October 2008.[4] Of course, the true test of our procedure is its forecasting performance, which is discussed in detail in the paper.

B.2 Economic Performance

Figures B-6–B-7 plot the time series of equity weights for the monthly portfolios based on the baseline and tilted densities for all predictors, along with the equity weights implied by the HA-SV benchmark densities. These figures contain the same information as Figure 5 in the paper for all 15 predictors considered.

While the HA-SV equity weights are relatively stable, oscillating between 0.25 and 0.5, the baseline and tilted equity weights exhibit more variation. The tilted weights are generally larger than the baseline and benchmark ones, which means that the tilted densities tend to imply larger equity positions. The various panels in Figures B-8 and B-9 show the corresponding log cumulative wealth for the three portfolios. By and large, the wealth generated by the tilted density forecasts lies above the baseline and benchmark counterparts, a pattern that is consistent throughout the whole out-of-sample period across the vast majority of the predictors, and in line with the certainty equivalent return gains reported in the paper.

C GARCH(1,1) Model: Additional Results

Tables C-1 and C-2 provide a breakdown that allows us to assess the stability of the statistical and economic performance of the GARCH(1,1) model as we did using tables 1 and 2 for the SV model in the paper. As it is the case in the paper, we use OOS-I to refer to the OOS period January 1990–December 2006 and OOS-II to refer to the OOS period January 2007–December 2014.

Regarding the statistical performance, tilting leads to improvements in R^2_{OOS} in 13 out of 17

[4] The variance risk premium (VRP) statistic in Bollerslev et al. equals the end-of-month VIX minus the realized volatility during the whole month, so their VRP statistic attains a large negative value in October 2008. On the other hand, as equation (22) of the revised manuscript shows, our VRP is a regression-based transformation of the end-of-month VIX only. Hence, because end-of-month VIX exceeded beginning of month VIX, our VRP statistic attains a large positive value in October 2008.

models in period OOS-I. For the models using NTIS and LTR as predictors, the R^2_{OOS} for the tilted densities is both positive and statistically significant at 5%. In the case of OOS-II, the improvements are limited to 9 models. We now see a positive and statistically significant at 5% R^2_{OOS} only for the model using RVOL as predictor. In the case of the ALSDs and OOS-I, the tilted densities outperform the baseline densities in all 17 models. With the exception of 4 models, the differences are statistically significant at either 5% (8 models) or 1% (5 models). In the case of OOS-II, the ALSDs for the tilted densities exceed their baseline analogs in 14 cases. The differences are now statistically significant at either 1% or 5% in 4 cases. Hence, tilting improves the statistical performance of the baseline densities for both OOS-I and OOS-II and more so for OOS-I.

Moving to the economic performance of the GARCH densities, the tilted CERDs are higher than the baseline CERDs for both OOS-I and OOS-II when $A = 3$ with one exception (INFL). The OOS-I baseline CERDs have an average of 1.032% while the tilted CERDs have an average of 2.233%. The higher average tilted CERDs are driven by the CERDs for models using predictors such as TMS (4.621%), TBL (4.145%), and DFR (4.265%). The OOS-II baseline (tilted) CERDs have an average of 0.559% (2.694%). Among the larger tilted CERDs for OOS-II are the ones associated with SII (8.787%), EP (5.951%), and EWC (5.633%). The tilted CERDs are also higher than the baseline CERDs for both OOS-I and OOS-II when $A = 5$ with one exception (RVOL). The average OOS-I baseline (tilted) CERD is 0.414% (1.470%). Some of the larger OOS-I tilted CERDs are those for TBL (2.753%), EP (2.489%), and TMS (2.403%). The average OOS-II baseline (tilted) CERD is 0.363% (2.324%) with SII (7.444%), DFR (4.851%), and EWC (4.534%), being among the main drivers of the higher average tilted CERD. Overall, tilting improves the economic performance of the baseline densities for both OOS-I and OOS-II and more so for OOS-II.

References

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Table C-1: Out-of-sample statistical predictability, GARCH(1,1) model

	Panel A: Out-of-sample R^2 (vs. HA-GARCH)					
	1990:01-2014:12		*1990:01-2006:12*		*2007:01-2014:12*	
	(1)	(2)	(3)	(4)	(5)	(6)
Model	*Baseline*	*Tilted*	*Baseline*	*Tilted*	*Baseline*	*Tilted*
DP	-0.426	**0.142**	-1.295	**-0.414**	0.952	**1.022**
DY	-0.484	**0.236**	-1.433	**-0.237**	1.018	0.985
EP	0.355	**1.031**	0.487	**0.725**	0.146	**1.515**
DE	-0.629	**0.334**	-0.313	**-0.276**	-1.130	**1.299**
RVOL	-0.004	**0.335**	-0.467	**-0.032**	0.729	**0.916****
BM	0.729	**0.735**	0.535	0.506	1.036	**1.097**
NTIS	-1.168	**-0.327****	-1.156	**0.170****	-1.186	**-1.114**
TBL	0.898	**0.906**	0.700	**0.721**	1.212	1.199
LTY	1.023	1.020	0.860	0.844	1.281	**1.299**
LTR	0.106	**0.291**	-0.304	**0.721****	0.755	-0.389
TMS	0.248	0.207	-0.115	-0.164	0.823	0.794
DFY	-1.120	**-0.312****	-1.429	**-0.027****	-0.629	-0.763
DFR	-0.081	-0.762	-1.755	**-0.610**	2.571	-1.002
INFL	0.453	**0.566**	1.037	**1.191**	-0.472	**-0.423**
SII	1.899	1.790	0.191	**0.401**	4.606**	3.989
KS	-22.973	**-14.158****	-18.535	**-3.998*****	-30.005	-30.253
EWC	0.954	0.937	0.681	0.630	1.386	**1.422**
	Panel B: Average log score differences (vs. HA-GARCH)					
	1990:01-2014:12		*1990:01-2006:12*		*2007:01-2014:12*	
	(1)	(2)	(3)	(4)	(5)	(6)
Model	*Baseline*	*Tilted*	*Baseline*	*Tilted*	*Baseline*	*Tilted*
DP	-0.093	**-0.002**	-0.062	**0.047*****	-0.158	**-0.105**
DY	-0.049	**0.042**	-0.043	**0.053*****	-0.060	**0.020**
EP	-0.040	**0.052***	-0.041	**0.063*****	-0.038	**0.028**
DE	-0.049	**0.018**	-0.044	**0.035**	-0.060	**-0.018**
RVOL	-0.061	**0.058*****	-0.042	**0.047****	-0.103	**0.082****
BM	-0.041	**0.046**	-0.036	**0.059****	-0.051	**0.019**
NTIS	-0.081	**0.001**	-0.039	**0.058****	-0.169	**-0.122**
TBL	-0.070	**-0.023**	-0.044	**0.030**	-0.124	-0.135
LTY	-0.046	**-0.001**	-0.046	**0.060****	-0.045	-0.129
LTR	-0.042	**0.016**	-0.041	**0.059****	-0.044	-0.078
TMS	-0.051	**0.016**	-0.043	**0.028**	-0.070	**-0.009**
DFY	-0.072	**0.058*****	-0.057	**0.045****	-0.105	**0.085****
DFR	-0.037	**0.068*****	-0.046	**0.056****	-0.019	**0.093*****
INFL	-0.036	**0.052**	-0.033	**0.065****	-0.043	**0.025**
SII	-0.044	**0.032**	-0.044	**0.057*****	-0.043	**-0.021**
KS	-0.188	**-0.018**	-0.235	**-0.004**	-0.089	**-0.047**
EWC	-0.034	**0.075*****	-0.038	**0.060*****	-0.026	**0.106*****

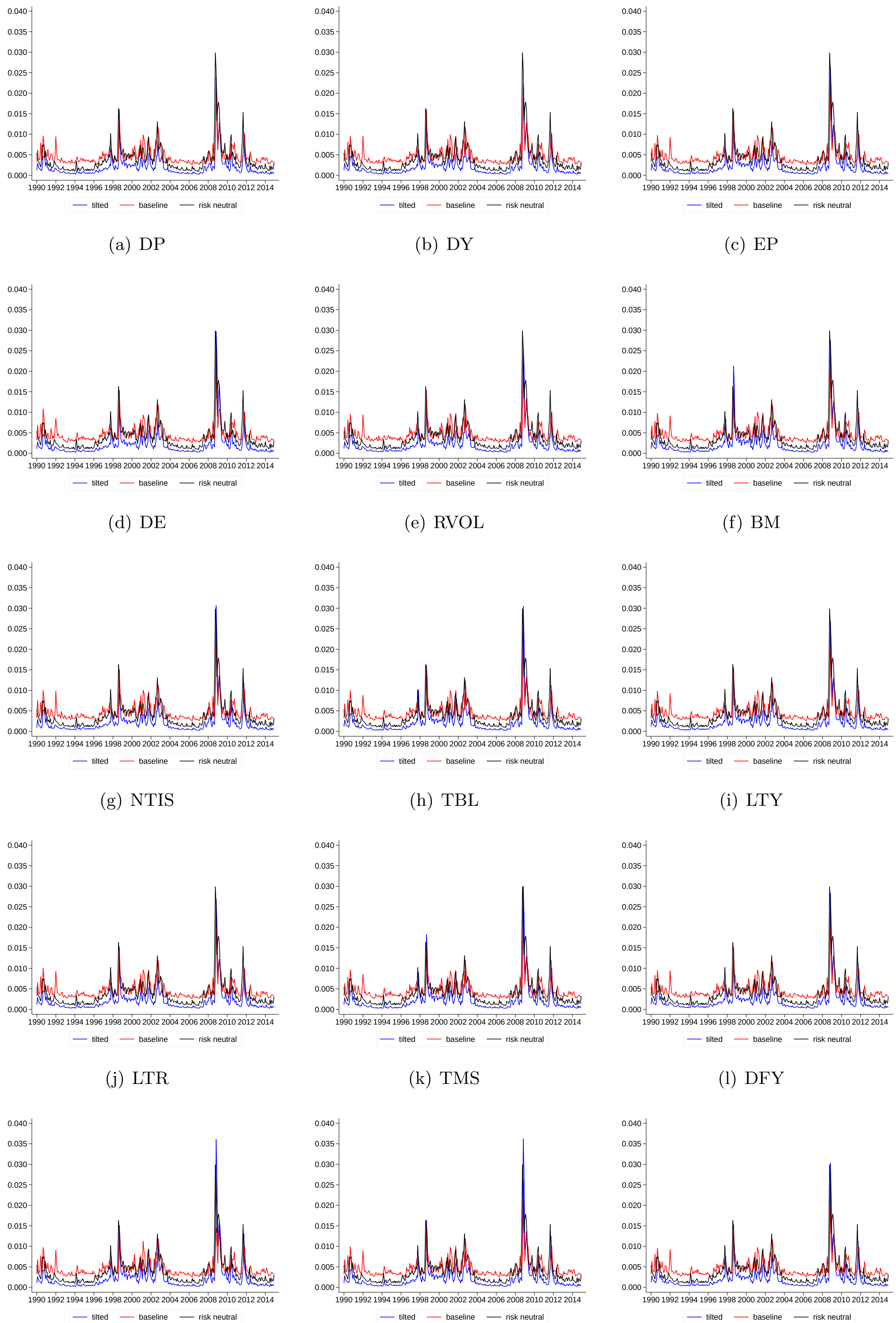
Note: The table reports the out-of-sample (OOS) R^2 and the average log score differences (ALSDs) for the 17 models considered, over the entire OOS period, 1990:01–2014:12, as well as for 1990:01–2006:12 and 2007:01–2012:14. All forecasts are OOS using recursive estimates for 1990:01–2014:12. Bold numbers indicate all instances where the tilted forecasts improve upon the corresponding baseline forecasts. The asterisks indicate statistical significance at 10%(*), 5%(**), and 1%(***) levels, using the Diebold and Mariano (1995) tests. The model nomenclature is as follows: (1) DP: Log dividend price-ratio; (2) DY: Log dividend yield; (3) EP: Log earning-price ratio; (4) DE: Log dividend-payout ratio; (5) RVOL: Excess stock return volatility; (6) BM: Book-to-market ratio; (7) NTIS: Net equity expansion; (8) TBL: Treasury bill rate; (9) LTY: Long-term yield; (10) LTR: Long-term return; (11) TMS: Term spread; (12) DFY: Default yield spread; (13) DFR: Default return spread; (14) INFL: Inflation; (15) SII: Short interest index; (16) KS: Kitchen Sink; (17) EWC: Equally Weighted Combination.

Table C-2: Out-of-sample economic predictability, GARCH(1,1) model

	Panel A: CER gains, $A = 3$ (vs. HA-GARCH)					
	1990:01-2014:12		*1990:01-2006:12*		*2007:01-2014:12*	
	(1)	(2)	(3)	(4)	(5)	(6)
Model	*Baseline*	*Tilted*	*Baseline*	*Tilted*	*Baseline*	*Tilted*
DP	-1.106	**0.563**	-1.988	**-0.440**	0.745	**2.671**
DY	-0.894	**0.743**	-1.774	**-0.178**	0.953	**2.675**
EP	1.757	**4.057**	1.279	**3.154**	2.752	**5.951**
DE	2.113	**3.972**	2.427	**3.604**	1.465	**4.737**
RVOL	0.024	**1.810**	-0.159	**0.272**	0.404	**5.072**
BM	1.435	**2.824**	1.807	**3.090**	0.671	**2.278**
NTIS	-0.200	**1.032**	0.910	**2.302**	-2.450	**-1.538**
TBL	1.853	**2.535**	3.285	**4.145**	-1.035	**-0.705**
LTY	1.968	**2.500**	2.617	**2.884**	0.641	**1.711**
LTR	0.181	**0.994**	1.374	**2.375**	-2.233	**-1.793**
TMS	1.182	**2.880**	2.589	**4.621**	-1.659	**-0.618**
DFY	-2.128	**1.184**	-1.180	**0.266**	-4.052	**3.110**
DFR	2.254	**4.560**	2.144	**4.265**	2.481	**5.172**
INFL	1.468	**1.632**	3.062	2.626	-1.740	**-0.389**
SII	3.456	**4.693**	1.348	**2.775**	7.975	**8.787**
KS	-0.618	**0.659**	-1.707	**-0.475**	1.674	**3.049**
EWC	1.962	**3.627**	1.505	**2.672**	2.914	**5.633**
	Panel B: CER gains, $A = 5$ (vs. HA-GARCH)					
	1990:01-2014:12		*1990:01-2006:12*		*2007:01-2014:12*	
	(1)	(2)	(3)	(4)	(5)	(6)
Model	*Baseline*	*Tilted*	*Baseline*	*Tilted*	*Baseline*	*Tilted*
DP	-0.827	**0.386**	-1.480	**-0.349**	0.526	**1.913**
DY	-0.659	**0.500**	-1.304	**-0.187**	0.679	**1.926**
EP	1.041	**3.011**	0.726	**2.489**	1.690	**4.091**
DE	1.116	**2.224**	1.429	**1.541**	0.478	**3.643**
RVOL	0.017	**0.905**	-0.030	-0.263	0.115	**3.352**
BM	0.589	**2.257**	0.635	**2.258**	0.493	**2.252**
NTIS	-0.871	**0.179**	0.175	**1.692**	-2.969	**-2.830**
TBL	1.412	**2.317**	1.850	**2.753**	0.522	**1.432**
LTY	1.151	**2.363**	1.201	**2.336**	1.049	**2.416**
LTR	0.365	**1.221**	1.257	**2.017**	-1.432	**-0.386**
TMS	0.644	**2.014**	1.152	**2.403**	-0.386	**1.223**
DFY	-1.328	**0.671**	-0.739	**0.209**	-2.519	**1.626**
DFR	0.632	**2.975**	0.385	**2.075**	1.140	**4.851**
INFL	0.945	**1.783**	1.596	**2.212**	-0.373	**0.912**
SII	2.706	**3.515**	0.601	**1.667**	7.203	**7.444**
KS	-1.595	**0.623**	-1.402	**0.387**	-1.987	**1.108**
EWC	1.294	**2.649**	0.982	**1.745**	1.936	**4.534**

Note: The table reports the annualized certainty equivalent return differences (CERDs) for portfolio decisions based on recursive out-of-sample (OOS) forecasts of excess returns. Each period, an investor with power utility and coefficient of relative risk aversion $A = 3$ (top panel) or $A = 5$ (bottom panel) selects stocks and T-bills based on a predictive density differing both by the model considered and the predictive density entertained (baseline or tilted). See the notes of Table C-1 for the model nomenclature. The equity weights are constrained to lie in the $[-0.5, 1.5]$ interval. All forecasts are OOS using recursive estimates of the models for 1990:01–2014:12. Bold numbers indicate all instances where CER gains for the tilted densities exceed the CER gains for the baseline densities.

Figure B-1: Physical and risk-neutral variance series, SV model



(a) DP (b) DY (c) EP

(d) DE (e) RVOL (f) BM

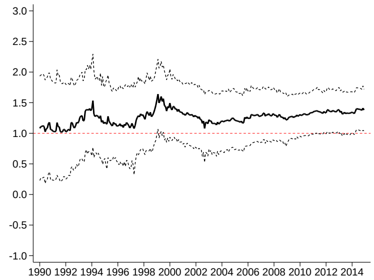
(g) NTIS (h) TBL (i) LTY

(j) LTR (k) TMS (l) DFY

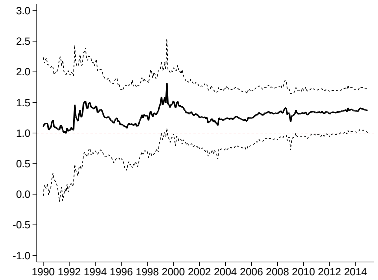
(m) DFR (n) INFL (o) SII

Figure B-2: Variance risk premium regression slope, SV model

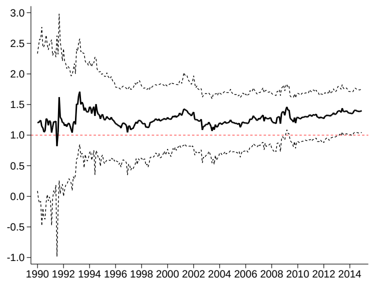
(a) DP



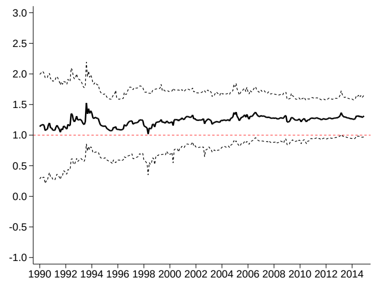
(b) DY



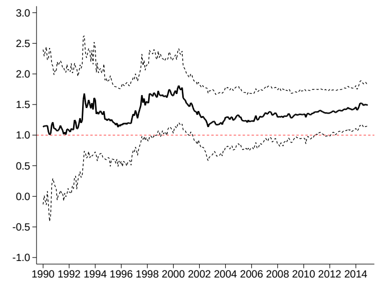
(c) EP



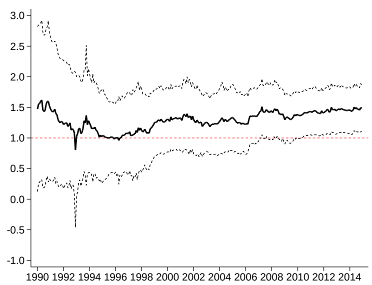
(d) DE



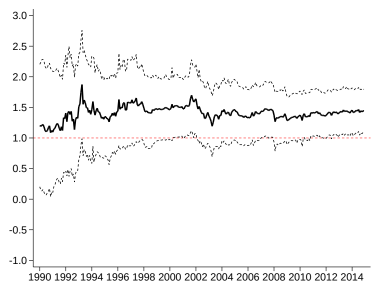
(e) RVOL



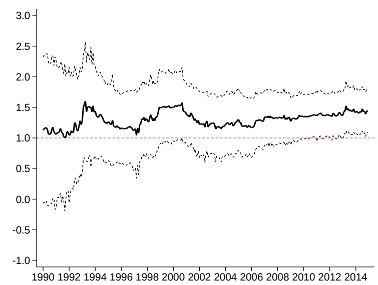
(f) BM



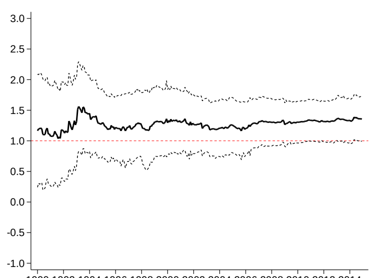
(g) NTIS



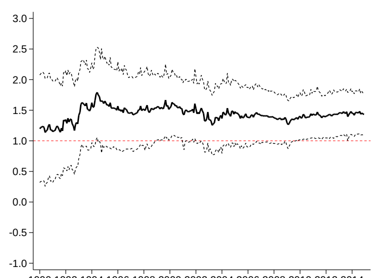
(h) TBL



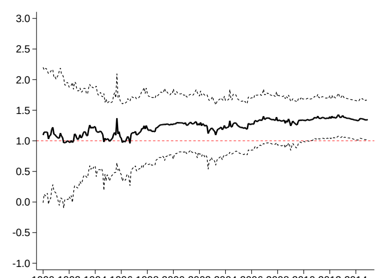
(i) LTY



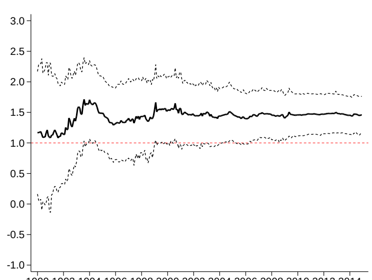
(j) LTR



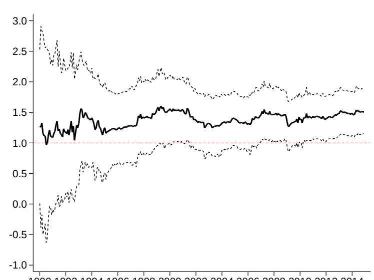
(k) TMS



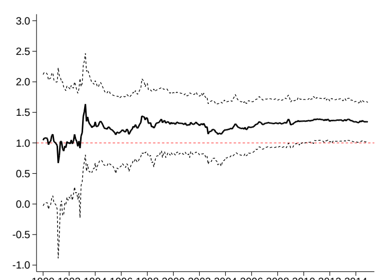
(l) DFY



(m) DFR

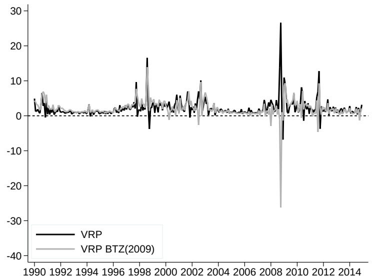


(n) INFL

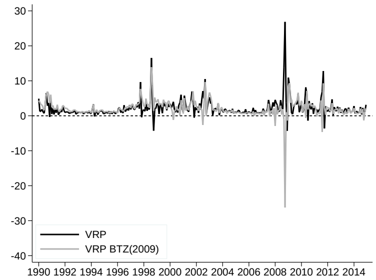


(o) SII

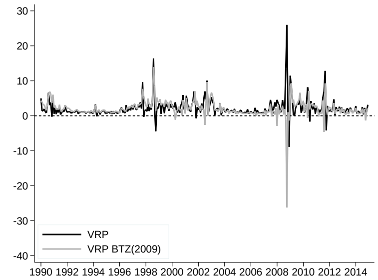
Figure B-3: Variance risk premium, SV model



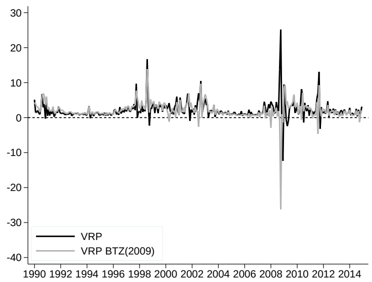
(a) DP



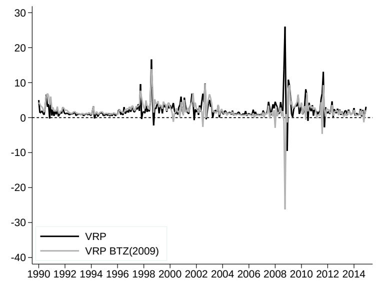
(b) DY



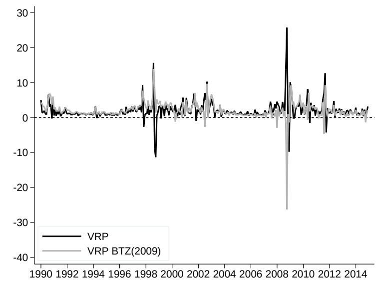
(c) EP



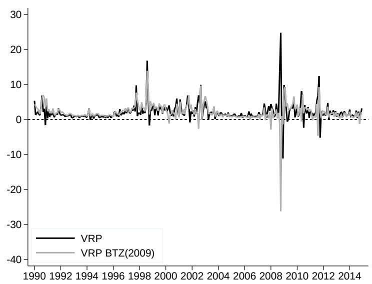
(d) DE



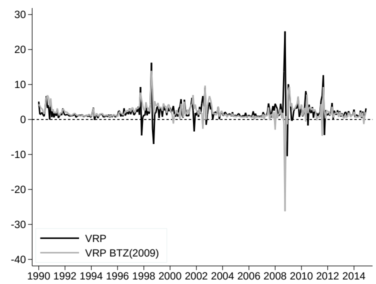
(e) RVOL



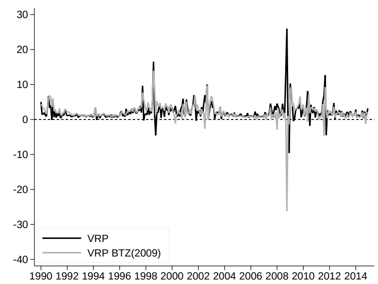
(f) BM



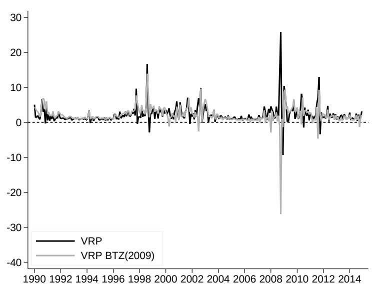
(g) NTIS



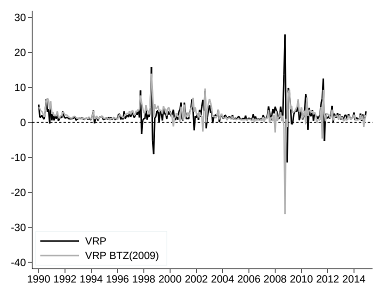
(h) TBL



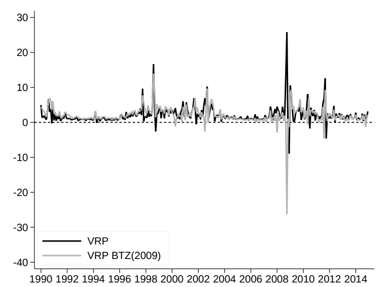
(i) LTY



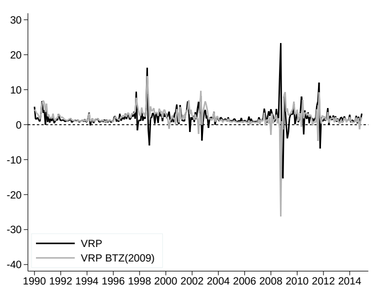
(j) LTR



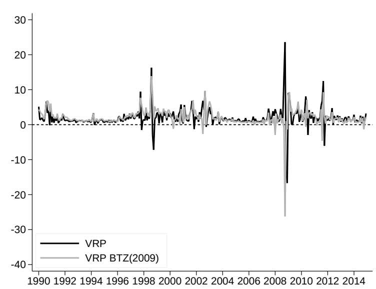
(k) TMS



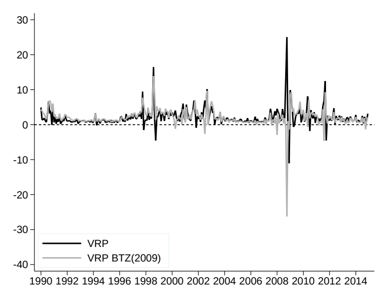
(l) DFY



(m) DFR

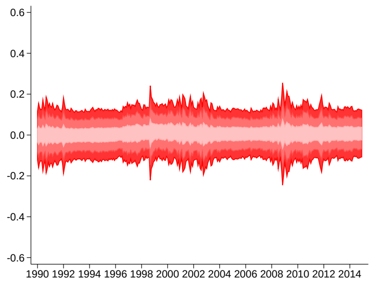


(n) INFL

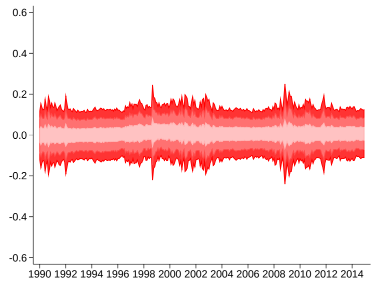


(o) SII

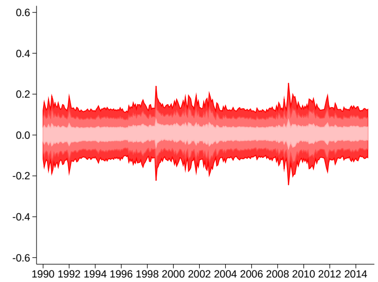
Figure B-4: Posterior probability intervals for baseline distributions, SV model



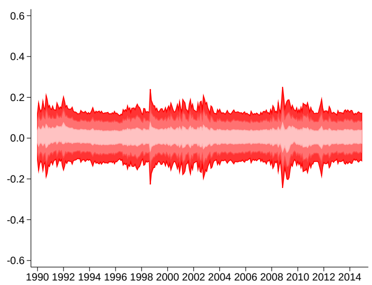
(a) DP



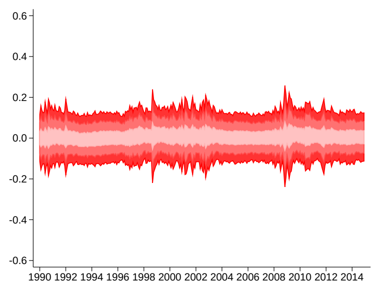
(b) DY



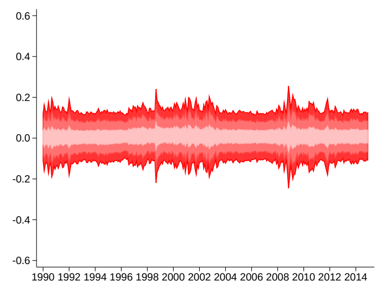
(c) EP



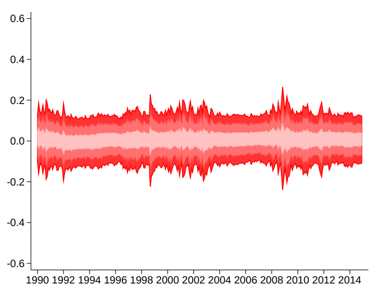
(d) DE



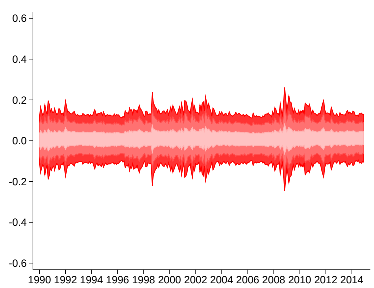
(e) RVOL



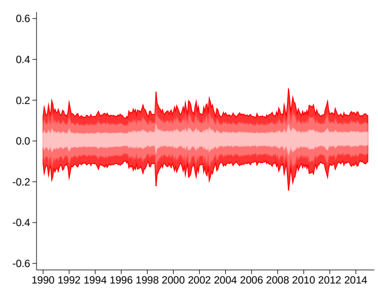
(f) BM



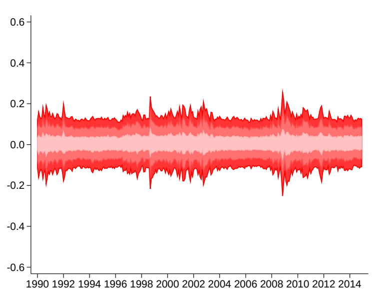
(g) NTIS



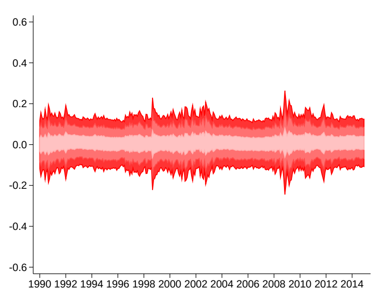
(h) TBL



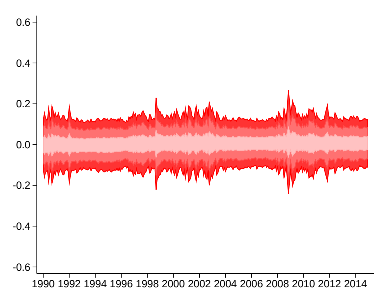
(i) LTY



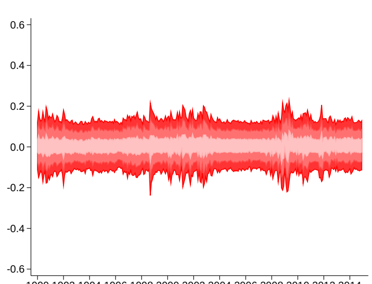
(j) LTR



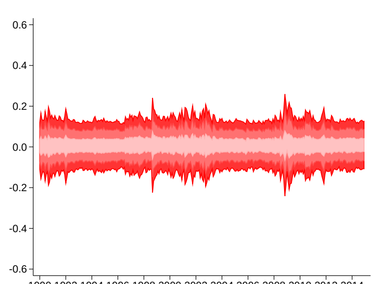
(k) TMS



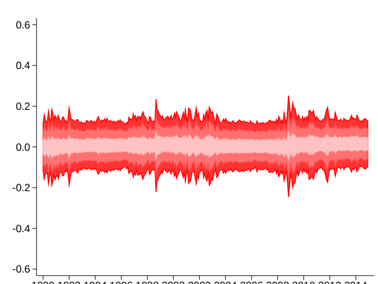
(l) DFY



(m) DFR

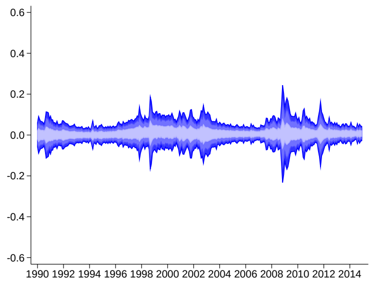


(n) INFL

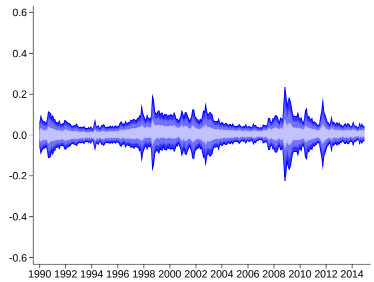


(o) SII

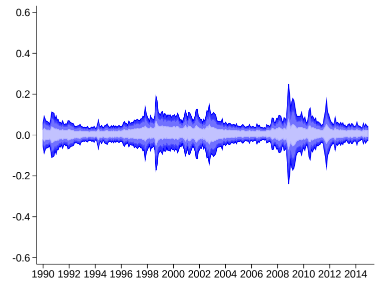
Figure B-5: Posterior probability intervals for tilted distributions, SV model



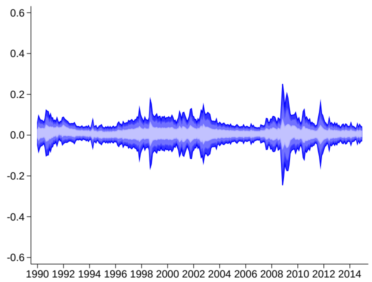
(a) DP



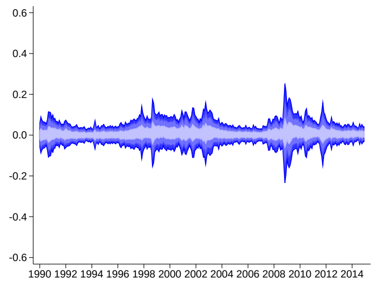
(b) DY



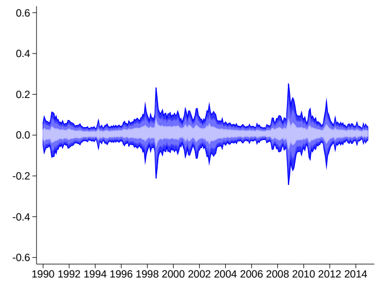
(c) EP



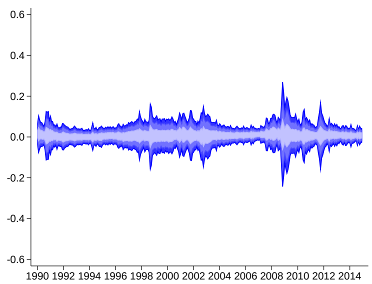
(d) DE



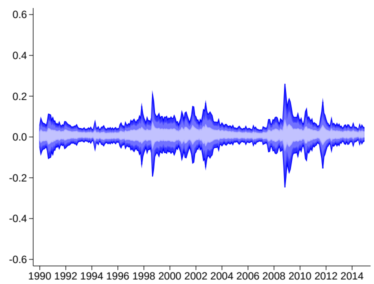
(e) RVOL



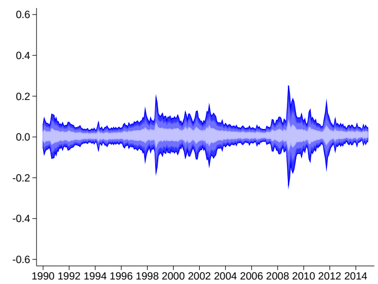
(f) BM



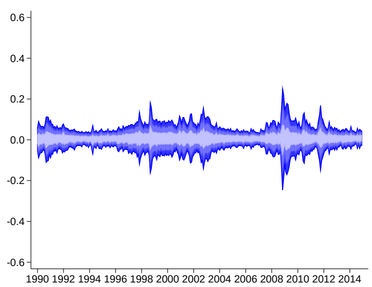
(g) NTIS



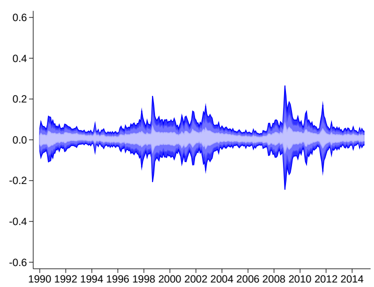
(h) TBL



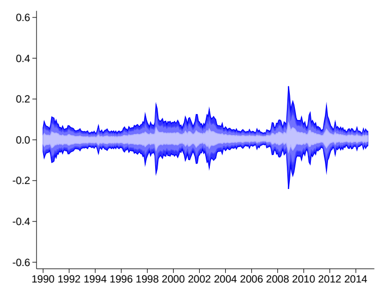
(i) LTY



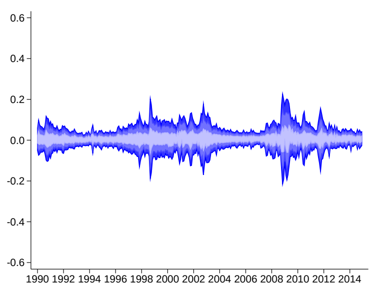
(j) LTR



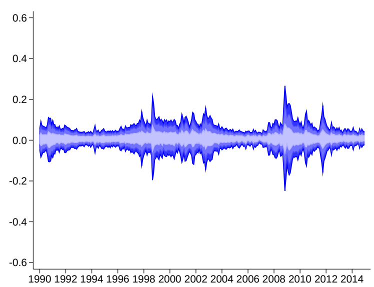
(k) TMS



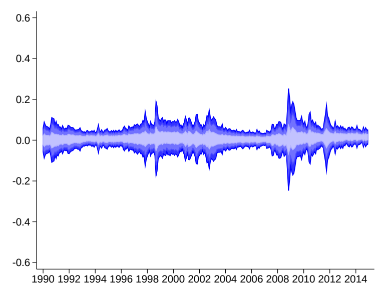
(l) DFY



(m) DFR

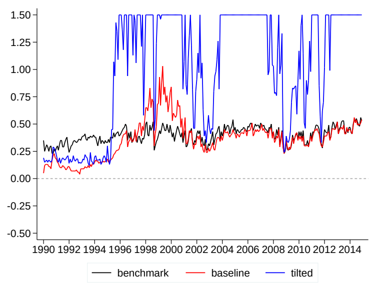


(n) INFL

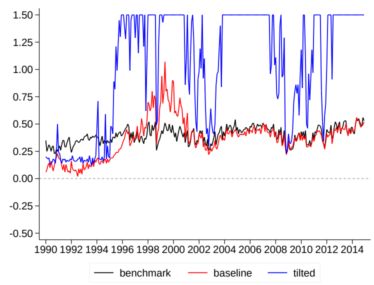


(o) SII

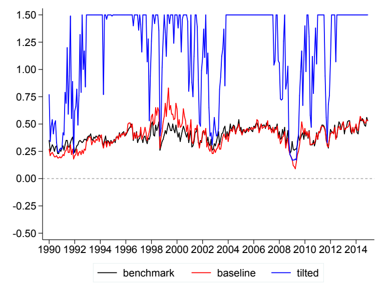
Figure B-6: Asset allocation weights, $A = 3$, SV model



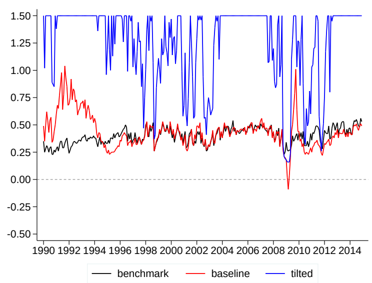
(a) DP



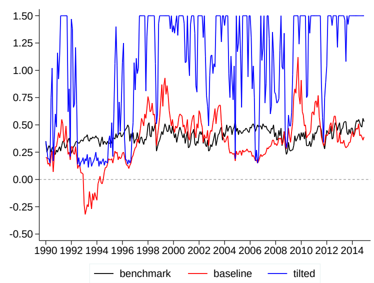
(b) DY



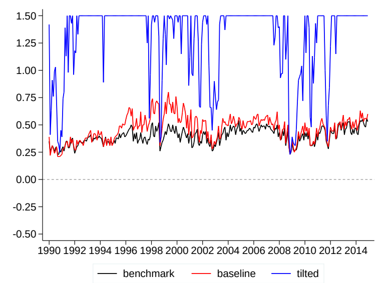
(c) EP



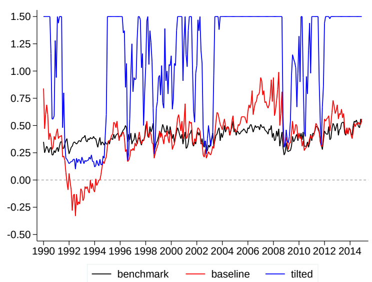
(d) DE



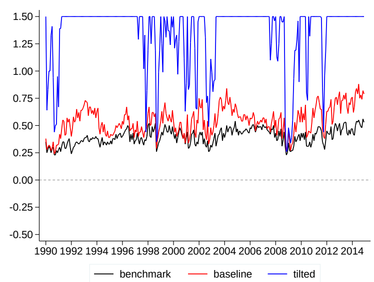
(e) RVOL



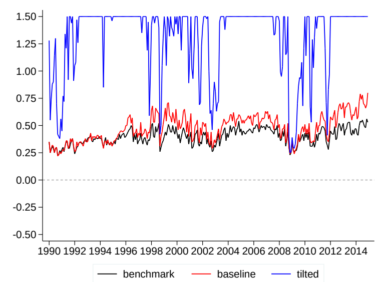
(f) BM



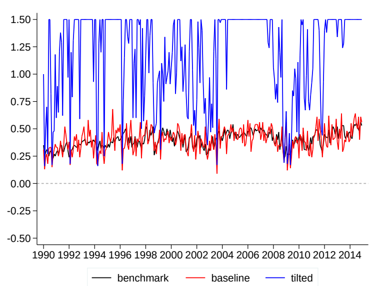
(g) NTIS



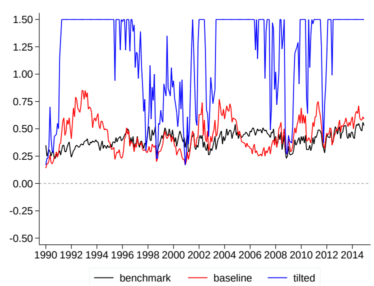
(h) TBL



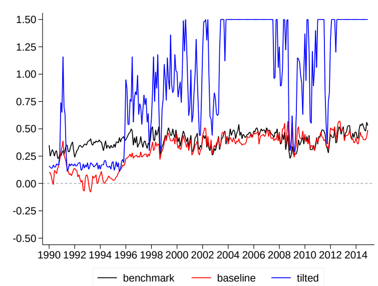
(i) LTY



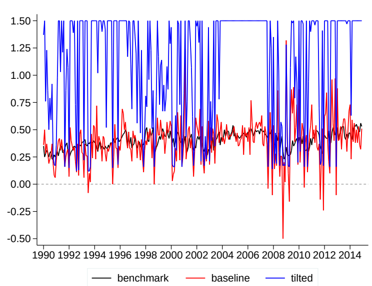
(j) LTR



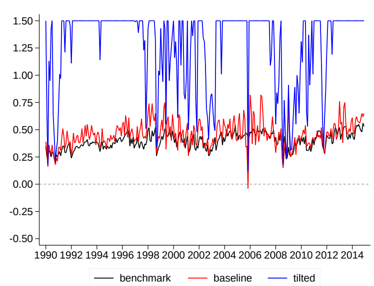
(k) TMS



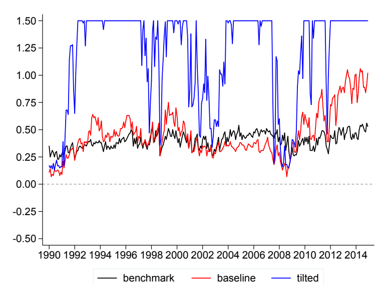
(l) DFY



(m) DFR

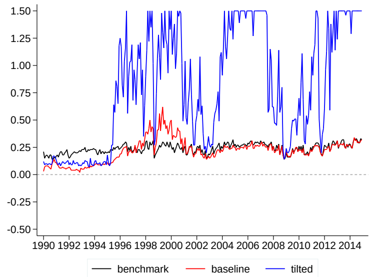


(n) INFL

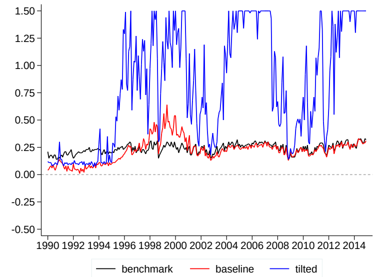


(o) SII

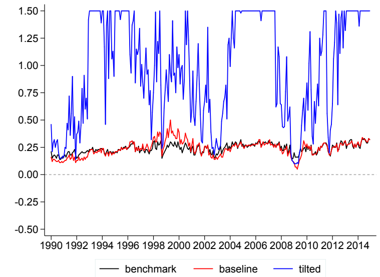
Figure B-7: Asset allocation weights, $A = 5$, SV model



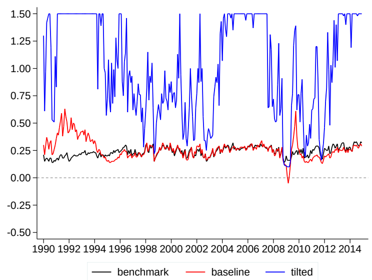
(a) DP



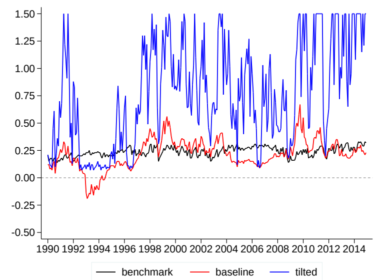
(b) DY



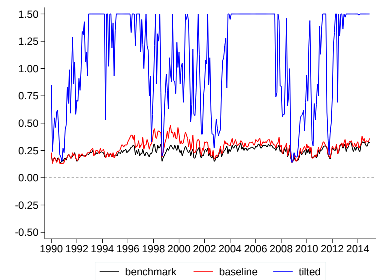
(c) EP



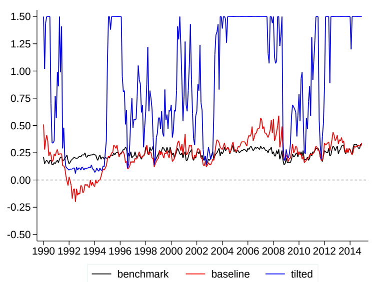
(d) DE



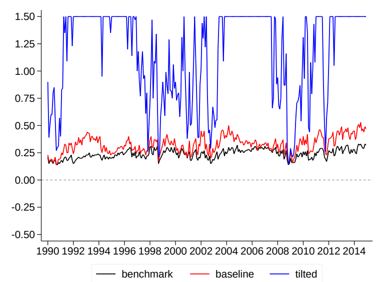
(e) RVOL



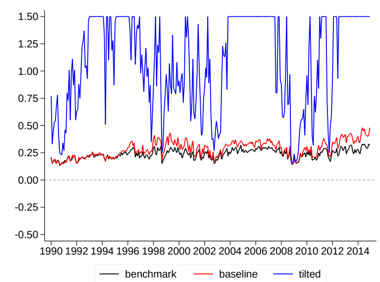
(f) BM



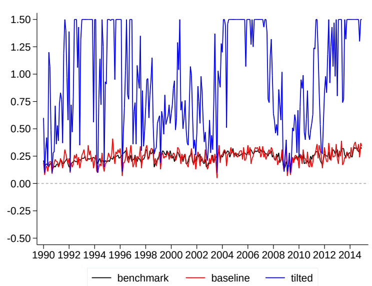
(g) NTIS



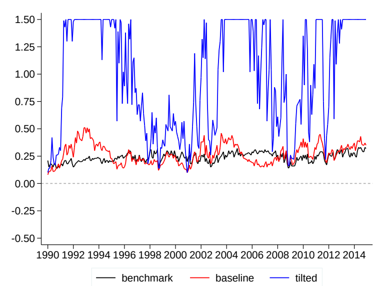
(h) TBL



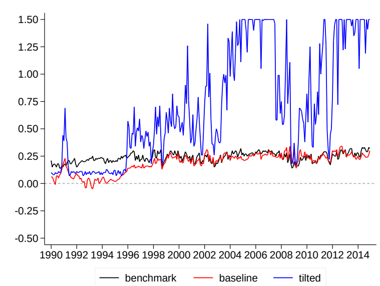
(i) LTY



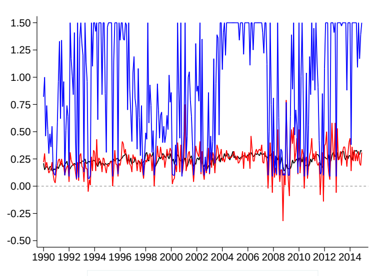
(j) LTR



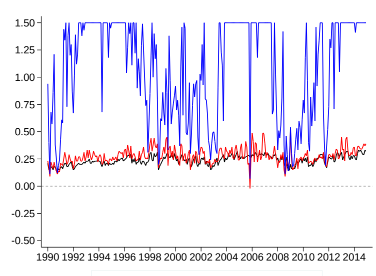
(k) TMS



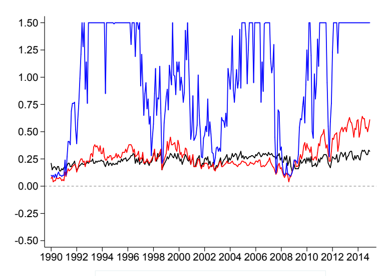
(l) DFY



(m) DFR

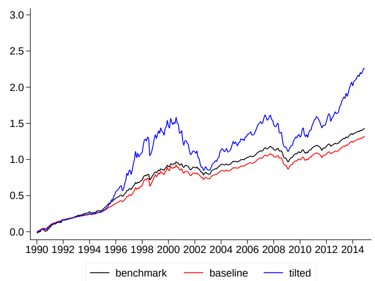


(n) INFL

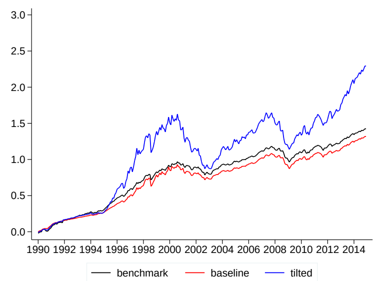


(o) SII

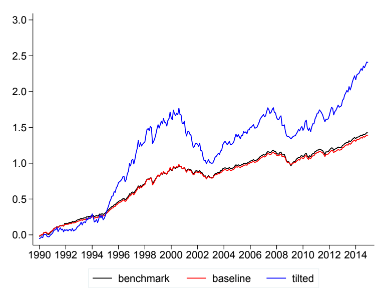
Figure B-8: Cumulative wealth, $A = 3$, SV model



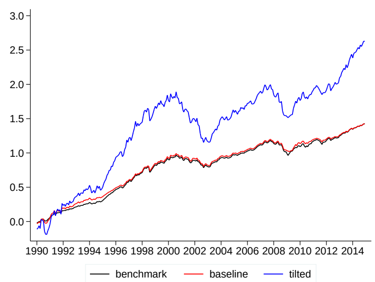
(a) DP



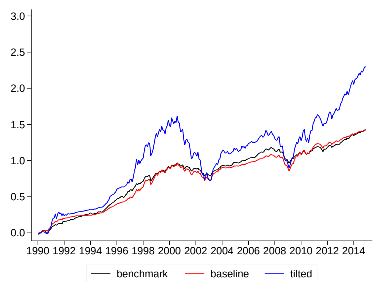
(b) DY



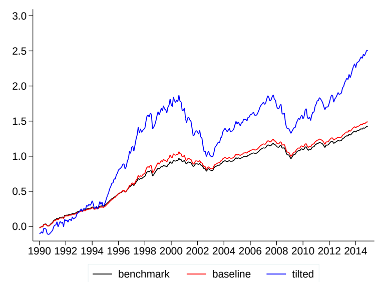
(c) EP



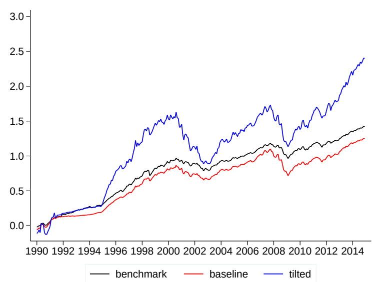
(d) DE



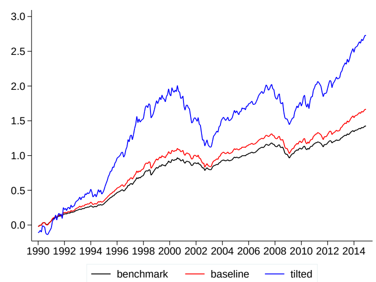
(e) RVOL



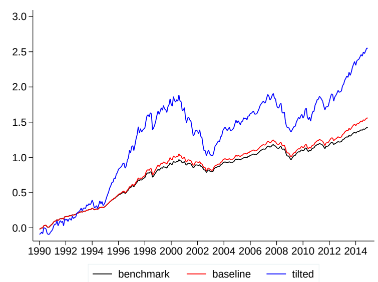
(f) BM



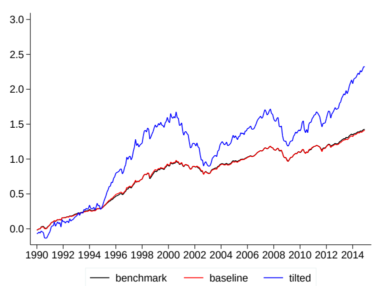
(g) NTIS



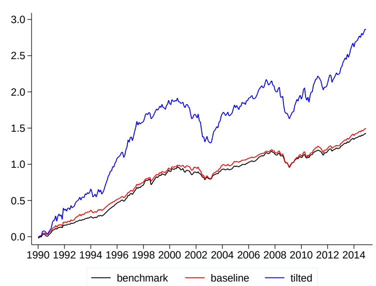
(h) TBL



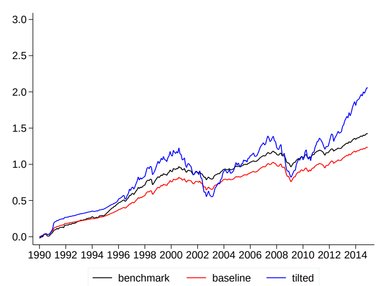
(i) LTY



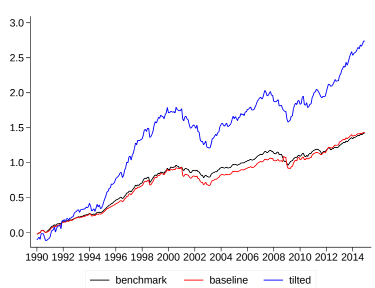
(j) LTR



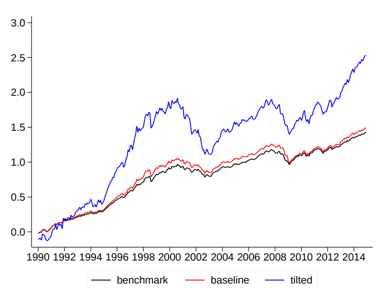
(k) TMS



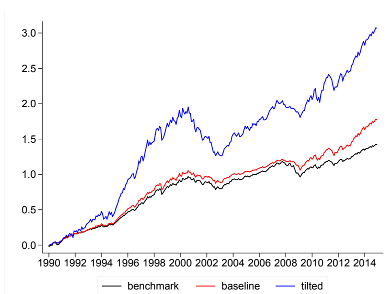
(l) DFY



(m) DFR

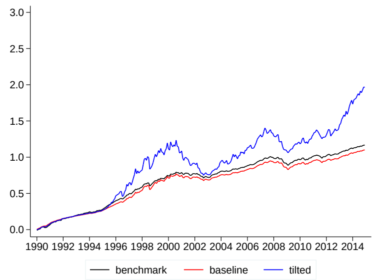


(n) INFL

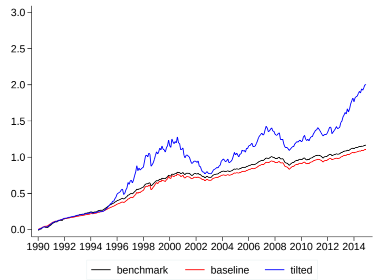


(o) SII

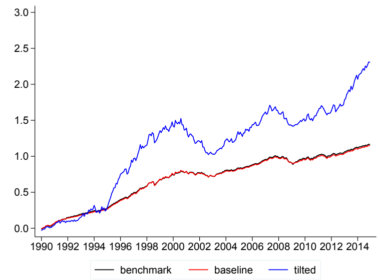
Figure B-9: Cumulative wealth, $A = 5$, SV model



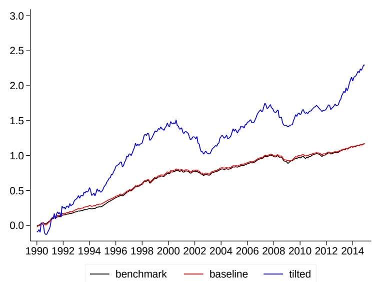
(a) DP



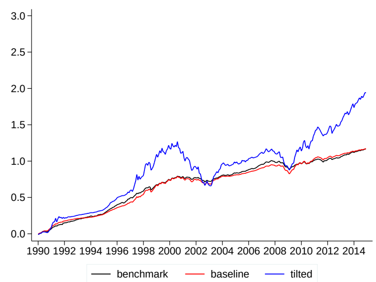
(b) DY



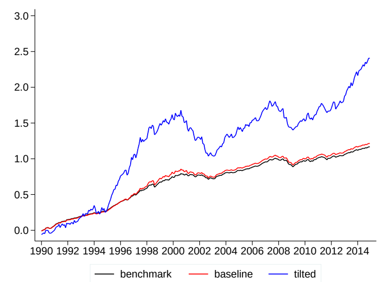
(c) EP



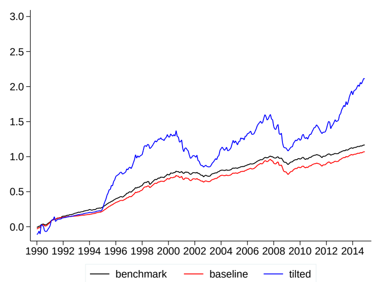
(d) DE



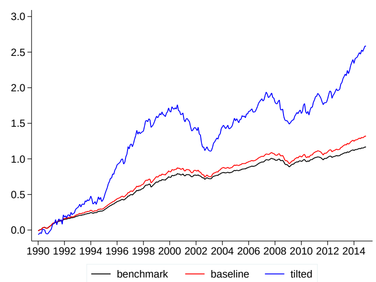
(e) RVOL



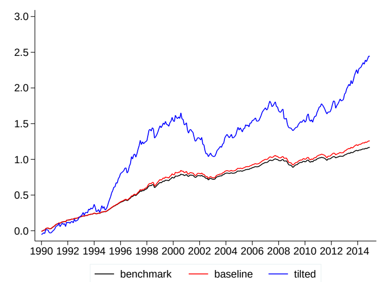
(f) BM



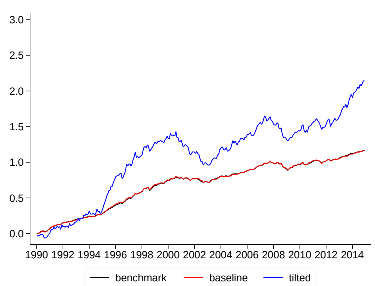
(g) NTIS



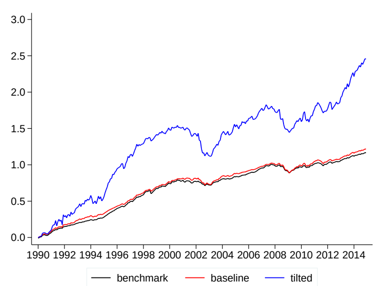
(h) TBL



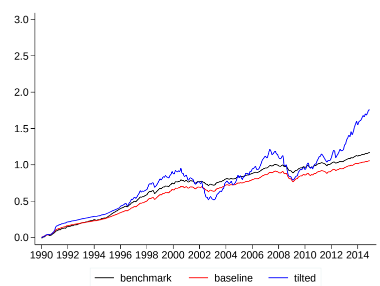
(i) LTY



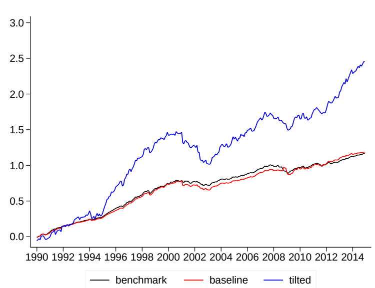
(j) LTR



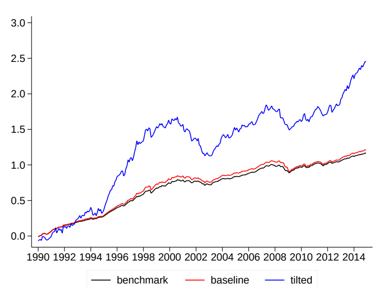
(k) TMS



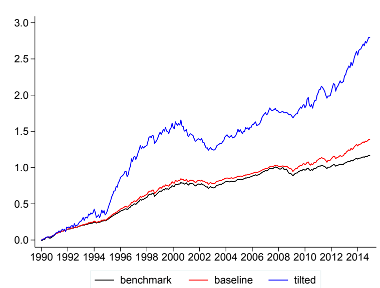
(l) DFY



(m) DFR



(n) INFL



(o) SII